1) What is the form of the particular solution to $y'''' + 18y'' + 81y = \cos(2t)$
   (A) $Y = At^2 \cos(2t) + B\sin(2t)$
   (B) $Y = At^2 \cos(2t) + Bt^2 \sin(2t)$
   (C) $Y = At^2 \cos(2t) + B\sin(2t) + C\cos(2t) + D\sin(2t)$
   (D) $Y = At \cos(2t) + B\sin(2t)$
   (E) $Y = A \cos(2t) + B\sin(2t)$

2) Which of the following sets of functions are linearly INDEPENDENT?
   i. $f(t) = 6t, g(t) = 3t^2 - 3, h(t) = 3t - 6$
   ii. $f(t) = 5t - 20, g(t) = 5t + 15, h(t) = 10t - 5$
   iii. $f(t) = 2, g(t) = 2t, h(t) = 2t^2$
   (A) i and iii
   (B) iii only
   (C) i only
   (D) ii only

3) The displacement $u(x,t)$ of an infinite string is described by the wave equation $\frac{1}{4}u_{xx} = u_{tt}$. We have $u(x,0) = 2(x-2)$ for $2 \leq x \leq 4, u(x,0) = 2(6-x)$ for $4 < x < 6$, and $u(x,0) = 0$ outside of $[2,6)$. Which of the following plots represents $u(x,4)$?
4) The temperature $u(x, t)$ in a bar of length 4 with heat diffusivity $\alpha^2 = \frac{1}{4}$ satisfies the heat equation $\alpha^2 u_{xx} = u_t$. If both ends of the bar are insulated and $u(x, 0) = 6$ for $0 \leq x \leq 1$ and $2$ for $1 < x \leq 4$, then evaluate the limit as $t \to \infty$ for $u(x, t)$.

- (A) 2
- (B) 6
- (C) 3
- (D) 0
- (E) none of these

5) Which of the following integrals are zero for all $L > 0$?

- (i) $\int_{-L}^{L} \sin(3t) e^{2t} dt$
- (ii) $\int_{-L}^{L} x^3 \cos x \, dx$
- (iii) $\int_{-L}^{L} t \sin(t^4) \, dt$
- (iv) $\int_{-L}^{L} |x^2 - 5| \sin x \, dx$

- (A) (i) only
- (B) all of these
- (C) (iv) only
- (D) (ii) only
- (E) (iii) only

6) The function $f(x) = \begin{cases} 1 - x, & -3 \leq x < 1, \\ 0, & x = 1, \\ -2x, & 1 < x < 3, \end{cases}$ is defined on the interval [-3,3].

The Fourier series for $f$ at $x = 1$:

- (A) converges to -3
- (B) converges to 1
- (C) converges to 0
- (D) does not converge
- (E) converges to -1

Free Response Questions:

7) Compute all the eigenvalues and corresponding eigenfunctions for the boundary value problem $y''' - \lambda y = 0$, $y'(-2) = 0$, $y(0) = 0$. If a certain range of the real numbers does not include any eigenvalues, show why there are none in that range.
8) Consider the function \( f(x) = 1 - x \) defined on the interval \( x \in [-1,1) \).

(a) Sketch the 2-periodic extension of \( f(x) \) on the interval \( x \in [-3,3] \).

(b) Compute the 2-periodic Fourier series representation of \( f(x) \).